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Counting Numbers by Centibel – A Suggestion

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Abstract

Analysing measurements of number concentration is quite unwieldy. Due to a range of several orders of magnitude one has to resort to the standardised decimal presentation, which means one has to keep in mind two numbers, numeral and exponent of ten. On the other hand, number concentration may be the only reasonable method with regard to ultrafine particles, i.e. for particles of diameter less than one micrometer. In order to facilitate grasping of measurement results we suggest a characterisation closely related to vibration and noise; for distinguishing and probably due to need when establishing auto- and cross-correlation functions in time series analyses we suggest introducing the notion of dust level by means of centibel: Def.: ***Dust level DL = 100 log (N/N$_{ref}$) cB (centibel)***

Introduction.

It deserves to be noted that ultrafine particles of elemental carbon, soot, of less than 1µm diameter - apart from various methods of defining this diameter - are an artificial man made product. They occur primarily in Diesel combustion engines due to their high injection pressure. Further, only in recent years methods of detecting them had been developed; hence, the medical knowledge regarding risks associated with these "things" could not be gathered at an earlier point of time. Engine engineers should accept the fact that other scientific disciplines, such medical science and measuring techniques in physics are subject to progress as well; consequently, there is no need or reason for a complaint of the sort "so many requirements have been met already, why a new one again?" On top of that, the "ultra-fines" may be the one quality to tell engine emissions from other sources.

The PMP program of the EU, carried out by a body of high level scientists using government laboratories of renown, arrived at the conclusion that number count concentration will be the only way of determining emission limits of ultra-fines, (see EURO 6). However, practically dealing with results of measurements, for instance analysing time series or comparing immissions at different locations is quite tedious, due to the wide range over several orders of magnitude. In similar cases engineering has resorted to logarithmic presentation, for instance in noise and vibration or in chemistry to determine the acidity, i.e. the pH-value. This method of logarithmic presentation is proposed herewith as well; however the basic requirement of first sight distinction from other logarithmic presentations should be satisfied as well.

Proposed Suggestion: Centibel, cB.

Noise level definitions are well known, their definition is repeated just for the sake of providing consistence with existing notions:

Noise level L def.: $\mathbf{L = 10 \log(p/p_{ref})^2}$ dB (= decibels)
based on pressure $p_{ref} = 2 .10^{-5}$ Pa [1 Pa = 1 Newton/m²]
based on power density $\mathbf{L = 10 \log (I/I_{ref})}$
with $I_{ref} = 10^{-11}$ watt/m² dB (= decibels)

This presentation is convenient because only one number is needed, what otherwise in standardised floating decimal would require two numbers, numeral and exponent of ten. Further, a linear scale ranging from one to hundred is easily perceptible.

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We note, the argument of a logarithm requires a number (without a dimension of physics); hence, a reference unit has to be defined as above. A suitable choice of this reference unit is essential for practicability, it should be standardised as in noise abatement (it is not standardised in vibration). Noise or vibration levels are usually given with two, at most three significant digits, the first digit indicating the magnitude (power of ten) and the second the numeral; for instance, a noise level L = 68,4 dB means 10^6 times antilog (84), i.e. 7; hence, 7.0×10^6 is obtained. In terms of physics

$$\mathbf{I = I_{ref} \times 10^{L/10} = 10^{-11} \times 7x10^6 = 7x10^{-5} \text{ watt/m}^2}$$

The above example illustrates our proposed presentation for a number concentration level.

We suggest the definition of a level for a number concentration in terms of centibels such that usual level are running up to one thousand at most, however, in contrast to noise and vibration there may be negative levels as well, for instance if the concentration is less than the defined reference (for instance characterising hospital surgery rooms) .

Dust level def.: **dL = 100 log (N_c/N_{ref})** centibels

For the time being we suggest a reference number concentration of just

$$\mathbf{N_{ref} = 100 \text{ particles/cm}^3 \qquad [cm^{-3}]}$$
$$\mathbf{= 10^5 \text{ particles/litre}}$$
$$\mathbf{= 10^8 \text{ particles/m}^3}$$

By looking at published data, for instance T. Johnson or Mayer (Brisbane) we may expect number variations of about 4-5 powers of a magnitude, say from 200 to 700 centibels; less than 200 may be regarded as hospital surgery room quality, ambient air say in cities like Biel/Bienne, where plenty of data exist, as the VERT test has been developed there exhibit definitively a dust number level varying between 200 and 300 centibels – definitively more than 100 and certainly less than 1000 particles per cm³ .

By defining centibels just in analogy to decibels a distinct notation is achieved, such that dust level is immediately recognised.

Average concentration and mean level.

May there be a stationary time series, or alternatively an average over some locations or even just addition of bandwidth, some simple rules of calculation are quite useful.

Consider a statistical sample of dust levels DL_i (i = 1,2,…,n), as defined above; accordingly, the arithmetic mean value is by definition

$$DL_m = \frac{1}{n}\sum_{i=1} DL_i \qquad (1)$$

The difference of each DL_i to the mean may be denoted by

$$\Delta L_i = DL_m - DL_i \qquad (2)$$

Consequently, the first order moment M_1 of ΔL_i, i. e. its mean or average is

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$$M_1 = \frac{1}{n}\sum_{i=1}^{n} \Delta L_i = 0 \qquad (3)$$

According to a basic theorem of probability theory a random variable is either completely described by means of the distribution function or by all its moments.
Second order moment or variance is obtained as

$$M_2 = \frac{1}{n}\sum_{i=1}^{n} \Delta L_i^{\ 2} = \sigma_{\Delta L}^{\ 2} \qquad (4)$$

Square root of the variance σ is also well known as standard deviation.
Third order moment, indicating obliqueness of the distribution is defined

$$M_3 = \frac{1}{n}\sum_{i=1}^{n} \Delta L_i^{\ 3} = \tau^3 \qquad (5)$$

Note: factor of obliqueness is defined

$$\rho = \frac{1}{n\sigma^3}\sum \Delta L_i^{\ 3}$$

For engineering applications approximations up to third order are usually sufficient.

Based on the above mentioned theorem, an approximate relation to determine number concentration average from dust levels is obtained as follows.

$$DL_i = 100 \log \frac{N_i}{N_{ref}} \qquad (6)$$

$$\frac{N_i}{N_{ref}} = 10^{\frac{DL_i}{100}} = anti \log \frac{DL_i}{100} \qquad (7)$$

Considering the identity $10 = e^{\ln 10}$ and using the alternate notation of the exponential function e^x = exp x , one arrives at

$$\frac{N_i}{N_{ref}} = \exp\left[\frac{Dl_i}{100} \ln 10\right] \qquad (8)$$

$$\frac{N_i}{N_{ref}} = \exp\left[\frac{DL_m + \Delta L_i}{100} \ln 10\right] \qquad (9)$$

To be split into two factors, the second one being developed into the power series of the exponential function and truncated after the third order:

$$\frac{N_i}{N_{ref}} = 10^{\frac{DL_{im}}{100}} \exp\left[\frac{\Delta l_i}{100}\ln 10\right] \quad (10)$$

Resorting to the definition of the arithmetic mean value applied now for number-concentrations

$$\frac{N_i}{N_{ref}} = \frac{1}{n}\sum_{i=1}^{n}\frac{N_i}{N_{ref}} \quad (11)$$

yields

$$10^{\frac{DL_{im}}{100}} \frac{\overline{N}}{N_{ref}} = \frac{1}{n}\sum_{i=1}^{n}\left(1+\frac{1}{11}n\Delta L_i+\frac{1}{21}n^2\Delta L_i^{\ 2}+\frac{1}{31}n^3\Delta L_i^{\ 3}+\ldots\right)=$$
$$=1+\frac{1}{2}n^2\sigma_{\Delta L}^{\ 2}+\frac{1}{6}n^3\tau^3 \quad (12)$$

and finally

$$\frac{\overline{N}}{N_{ref}} = \left(1+\frac{n^2}{2}\sigma_{\Delta L}^{\ 2}+\frac{n^3}{6}\tau_{\Delta L}^{\ 3}\right) anti\log\frac{DL_m}{100} \quad (13)$$

Relation (13) provides a method to determine the mean concentration by means of the first three statistical moments of the dust levels.

If due to some reasonable hypothesis a probability distribution is selected and fit to the moments obtained, predictions of dust levels based on probabilistic considerations are feasible, which may be quite important for administrative measures, for instance traffic restrictions.

Time series of dust levels

Dust levels may be measured at the very same location in equal intervals of time, this way a time series is obtained. It is reasonable to assume this time series to be more or less stationary; hence, equ. (13) yields information, whether a certain level will be exceeded or not.

Further, the random variable, dust level $DL(t_i)$, as a function of time, may assume say k-states DL_k, (k = 1,…….m), finite discrete states. At time t_i the state of the dust level may be D_σ while at time t_{i-1} it had been D_β.

Obviously due to elementary physics, the dust level at time t_i depends only on the dust level at time t_{i-1} **and** the transition probability from state D_β το D_σ, the probability $p(\sigma|\beta)$ of state σ at time t_i conditional that at time t_{i-1} the state had been β, which is the classical definition of the discrete Markov process.

Selecting some suitable probability distribution, for instance a Poisson-d.), a Markov-Poisson process may be obtained. This fact in mind, there is a variety of mathematical tools around for extracting more information and more precisely from measurements available. Transition probabilities of Markov-processes are determined by diffusion coefficients, which will be dealt separately.

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